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INCREASING THE ACCURACY OF MEASUREMENT OF THE THERMAL CONDUCTIVITY OF FLUIDS

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Within the framework of the method of measurement in a stage of irregular thermal conditions it is suggested to impart a helical shape to a heat source (filament). An analysis of the influence of the source curvature is made and the results of investigation of binary solutions are presented.

The accuracy of measurement of thermal conductivity can be determined similarly to [1] in the form of $\lambda/\Delta\lambda$, where λ is the thermal conductivity of the substance investigated; $\Delta\lambda$ is the minimum change in the thermal conductivity which can be recorded (resolved) by a measuring device. The high precision of devices or instruments used to measure the thermal conductivity is of principal significance in considering fine effects associated, for example, with the influence of fields, for investigation of thermal conductivity in the vicinity of the phase conversion point, and in the study of solutions.

When applying the method of measuring thermal conductivity in a stage of irregular regime (the nonstationary method of a heated filament) usually a linear heat source (filament) is used which simultaneously serves as a heat receiver (resistance thermometer). Elementary calculations for the case of bridge circuit, into one arm of which a resistance thermometer is included, show that a useful signal from the bridge diagonal is equal to

$$\Delta U = \frac{U_0}{4}\,\alpha\Delta T\,, \tag{1}$$

where U_0 is the voltage supplied to the bridge; ΔT is the temperature increment due to the effect of the heating pulse. It is assumed that the bridge is symmetrical, i.e., the ratio of the resistances of semi-arms is equal to unity. Since $\Delta T \approx q_l$ [2], we have

$$\Delta T = \text{const}\,\frac{U_0^2}{l^2}\,. \tag{2}$$

With allowance for (1) and (2) we obtain that the signal ΔU is proportional to U_0^3.

The measurement procedure presupposes in the beginning the selection of the magnitude of heating in the pulse (ΔT = const). The parameter determines the selection of voltage U_0: $U_0 = \text{const}\ \Delta T^{1/2} l = \text{const}' l$. From this it follows that the value of the reference voltage U_0, as well as respectively the useful signal ΔU and precision, depend on the filament length: the longer the filament, the more favorable, in the sense of resolution, are the conditions for measurements: $\Delta U \sim U_0^3 \sim l^3$.

The organization of an experiment, along with the selection of the filament length, also presupposes account for such factors as the size of the cell and liquid volume. Moreover, it is advisable to reduce their number. This is possible, in particular, when the filament is placed in the cell more compactly, say in the form of a spiral.

In order to study the influence of the source curvature on the result of measurement, we will consider a model problem of nonstationary heating by a constant heat source flux which has the shape of a circle of radius R and is immersed into an infinite medium. A similar problem was considered in [3]. The asymptotics of the solution ob-

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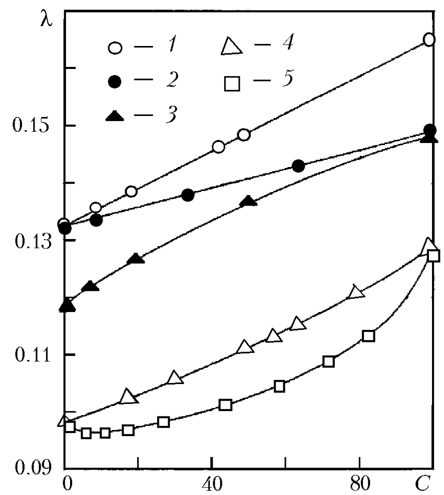

Fig. 1. Thermal conductivity of solutions: 1) dimethyl acetamide in toluene; 2) methyl ethyl ketone in toluene; 3) butyl alcohol in hexane; 4) toluene in butyl bromide; 5) toluene in CCl_4. λ, W/(m·K); C, %.

tained is obvious. The case of $R \to 0$ corresponds to heating of a point source, whereas that of $R \to \infty$, to heating of a linear source. As is known [4], the distribution of the temperature field created by the linear source has the form

$$T_{\text{lin}} = \frac{q_l}{4\pi\lambda} E_1\left(\frac{r^2}{4at}\right), \tag{3}$$

where q_l = const is the heat flux per unit length of the source. Expression (3) is valid for small values of the parameter $r^2/(4at)$.

To study the influence of the source curvature on distribution (3) it is necessary to consider the case of arbitrary values of R. In accordance with [3], the temperature at the interface between the source of radius r_0 and the medium is equal to

$$T_{\text{cur}} = \frac{q_l}{4\sqrt{\pi}\,\lambda}\sqrt{\alpha_1}\int_1^{\infty}\frac{\exp\left(-\left(\alpha_2+\alpha_3+\frac{\alpha_1}{2}\right)\right)I_0\left(\alpha_3+\frac{\alpha_1}{2}\right)\tau}{\sqrt{\tau}}\,d\tau\,, \tag{4}$$

where $\alpha_1 = \frac{R^2}{at}$; $\alpha_2 = \frac{r_0^2}{4at}$; $\alpha_3 = \sqrt{\alpha_1}\,\alpha_2 = \frac{Rr_0}{2at}$. The dimensional parameters entering into Eq. (4) in the experiment conducted had the following values: $a = 10^{-8}$ m^2/sec, $r_0 = 2\cdot10^{-5}$ m, $R = 5\cdot10^{-3}$ m, and $t = 5\cdot10^{-3}$ sec. Then $\alpha_2 = 2\cdot10^{-2}$ and $\alpha_1 = 5\cdot10^{-5}$.

Using the function

$$f(\alpha_1, \alpha_2, \alpha_3) = \sqrt{\alpha_1}\int_1^{\infty}\frac{\exp\left(-\left(\alpha_2+\alpha_3+\frac{\alpha_1}{2}\right)\right)I_0\left(\alpha_3+\frac{\alpha_1}{2}\right)\tau}{\sqrt{\tau}}\,d\tau$$

and relation (4), we obtain

$$T = T_{\text{lin}}\frac{\sqrt{\pi}\,f(\alpha_1, \alpha_2, \alpha_3)}{E_1(\alpha_2)}\,.$$

The calculation shows that the factor $\frac{\sqrt{\pi}\, f(\alpha_1, \alpha_2, \alpha_3)}{E_1(\alpha_2)}$ which reflects the curvature of the heat source under the experimental conditions is not less than 0.99, i.e., the difference between the temperature values due to this curvature did not exceed 1%.

Experiments were carried out according to the procedure given in [5]. As heat sources we used platinum springs. The technology of their production included the following operations: a) winding of a copper filament with a platinum core (of radius $2 \cdot 10^{-6}$ m) around a glass rod; b) etching of the copper shell by nitric acid; c) annealing of the platinum filament; d) removal of the glass rod. The step of the spring was $\sim 10^{-3}$ m; the length of the filament from which the spring was made was $\sim 10^{-1}$ m. Calculations show that when the liquid is heated to a maximum (by 4^{o}C) at the end of the pulse of length $5 \cdot 10^{-3}$ sec, the useful signal amounts to about 1 V.

Since the recording of the useful signal was performed by a variant of the null method and an oscillograph with the value of a division of 10^{-4} V was used as an indicator, the accuracy of measurements in the experiments carried out was on the order of 10^{4}. This means that the measuring circuit records a change in heat conduction at a level of hundredths of a percent. The error of relative measurements was estimated to be 3%. Both the errors due to the inadequatecy of the real measuring model and instrument errors were taken into account.

In order to test the proposed method of increasing the accuracy of measurements we studied the thermal conductivity of several binary systems. In particular, the dependences of the thermal conductivity of solutions of methyl ethyl ketone and dimethyl acetamide in toluene, as well as a solution of butyl alcohol in hexane on the volume concentration, are close to linear ones (see Fig. 1). However, for the carbon tetrachloride–toluene system, deviations from the additivity in the cocnentrational dependence are rather noticeable. Moreover, in the region of low concentrations of toluene a minimum is observed. This means that addition of a component having higher thermal conductivity makes the thermal conductivity of the solution lower. This minimum is poorly defined and can be detected only by a highly accurate method. We note that for a solution of toluene in butyl bromide whose thermal conductivity is close to that of carbon tetrachloride, experimental data depict a virtually linear character of the dependence of the thermal conductivity of the solution on the concentration ratio of components. The results given show that relations of additive character are not always correct.

In conclusion we note that compact disposition of a linear source (in the form of a spiral, as is done in the present work, or as a volute on backing) is promising as it increases the accuracy of thermal conductivity measuring methods. At the same time, it should be emphasized that this refers in the first place to short-term methods when the effect of the reference temperature drift is eliminated practically completely.

NOTATION

a, thermal diffusivity, m^2/sec; C, volume concentration, %; E_1, exponential integral; I_0, modified Bessel function of the first kind; l, length of the source (filament), m; q_l, heat flux per unit length of a linear source, W/m; R, radius of a source made in the form of a ring, m; r_0, radius of a source made in the form of a filament, m; r, distance from the source, m; T, temperature, K; t, time, sec; U_0, voltage supplied to the bridge, V; α, temperature coefficient of resistance, 1/K; λ, thermal conductivity, W/(m·K); τ, integration variable. Subscripts: lin, linear; cur, curvature; 0, initial value.

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